[5]Case, K., "Stability of Inviscid Plane Covette Flow," *Physics of Fluids*, Vol. 3, 1960, pp. 143-148.
[6]Landahl, M.T., "Wave Breakdown and Turbulence," *SIAM Journal of Applied Mathematics*, Vol. 28, 1975, pp. 735-756.
[7]Whitham, G.B., *Linear and Nonlinear Waves*, John Wiley and Sons, New York, 1974.
[8]Chin, W.C., "Effect of Dissipation and Dispersion on Slowly Varying Wavetrains," *AIAA Journal*, Vol. 18, Feb. 1980, pp. 149-158.

H80-014

Jet Thickness in High-Speed Two-Dimensional Planing

20006
20007

Peter R. Payne*
Payne, Inc., Annapolis, Md. 50001

Nomenclature

C_R $= R/\frac{1}{2}\rho u_0^2 l$
D_j = jet resistance
D_w = wave resistance per unit width
F_l = Froude number $= u_0/\sqrt{gl}$
g = acceleration due to gravity
l = wetted length
L = lift force per unit width $= R\cos\tau$ for a flat plate
R = resultant force per unit width
u_0 = freestream velocity
δ = jet thickness
ρ = mass density of the fluid
τ = trim angle
β = angle of the resultant force R aft of a normal to the undisturbed flow ($\beta = \tau$ for a flat plate)
θ = angle of the jet above the horizontal for a non-planar plate ($\theta = \tau$ for a flat plate)

PAYNE[3] has shown that the high-speed wave drag of any two-dimensional pressure field is, per unit width

$$D_w = (L^2/l^2)\cdot(1/\rho g)\cdot(1/F_l^4) \tag{1}$$

The jet drag[4] is, in the two-dimensional case

$$D_J = \rho u_0^2 \delta(1+\cos\tau) \tag{2}$$

Now for a flat plate in inviscid flow

$$D_w + D_J = R\sin\tau \tag{3}$$

Substituting for D_w and D_j from Eqs. (1) and (2), and writing $L = R\cos\tau$

$$\rho u_0^2 \delta(1+\cos\tau) = R\sin\tau - \frac{R^2\cos^2\tau}{l\rho g F_l^4}$$

$$\therefore \frac{\delta}{l} = \frac{C_R}{2(1+\cos\tau)}\left[\sin\tau - \frac{C_R\cos^2\tau}{2F_l^2}\right] \tag{4}$$

Received March 3, 1980; revision received April 11, 1980.
Index categories: Hydrodynamics; Jets, Wakes, and Viscid-Inviscid Flow Interactions; Marine Hydrodynamics, Vessel and Control Surface.
*President.

In the limit $F_l \to \infty$

$$(\delta/l) \to [C_R\sin\tau/2(1+\cos\tau)] = (C_R/2)\tan(\tau/2)$$

which is a form of Pierson and Leshnover's[2] Eq. (3), derived from both momentum theory and their exact solution of the flowfield. The additional terms in Eq. (4) (which reduce the jet thickness at finite Froude numbers) are due to the heavy fluid effect of wave drag.

Squire's[1] high-speed theory result is[5]

$$\frac{\delta}{l} \simeq \frac{\pi}{4}\tau^2 \Big/ \left[1 + \frac{5}{8}\frac{\pi}{F_l^2}\right]^2 \tag{5}$$

based upon

$$C_R \simeq \pi\tau \Big/ \left[1 + \frac{5}{8}\frac{\pi}{F_l^2}\right] \tag{6}$$

If we substitute Eq. (6) for C_R into Eq. (4) we obtain Eq. (5), but multiplied by the factor

$$[1 + (\pi/8F_l^2)]$$

Thus, for both theories

$$\frac{\delta}{l} \to \frac{\pi\tau^2}{2(1+\cos\tau)} \quad \text{as} \quad F_l \to \infty \tag{7}$$

which expresses the fact that, in the limit, all the resistance is due to jet drag.

For this limit, Eq. (7) is compared in Fig. 1 with the Schwarz-Christoffel transformation solution of Pierson and Leshnover,[2] who obtained

$$\frac{\delta}{l} = \pi\left[\frac{1+\cos\tau}{1-\cos\tau} - \log\left(\frac{1-\cos\tau}{2\cos\tau}\right) + \frac{\pi\sin\tau}{1-\cos\tau}\right]^{-1} \tag{8}$$

The agreement between Eqs. (7) and (8) is good for small trim angles, but the theories diverge above about $\tau = 5$ deg.

This is because Pierson and Leshnover obtain a normal force coefficient for a plate planing on a weightless fluid

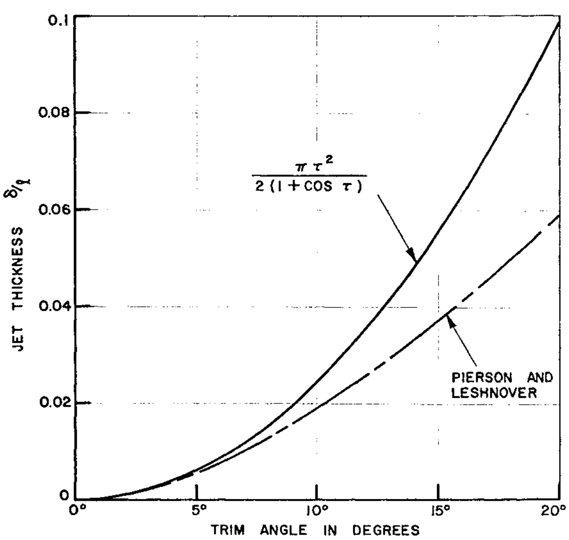

Fig. 1 Jet thickness as function of trim angle for limit of infinite Froude number. Pierson-Leshnover theory is for two-dimensional planing on weightless fluid.

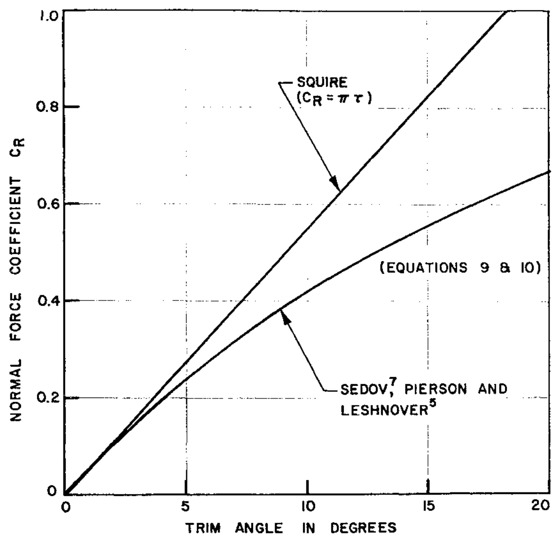

Fig. 2 Normal force coefficient for flat plate, as function of trim angle, in limit $F_l \to \infty$.

($F_l \to \infty$) of

$$C_R = \frac{2\pi\sin\tau}{1+\cos\tau-(1-\cos\tau)\log\left(\frac{1-\cos\tau}{2\cos\tau}\right)+\pi\sin\tau} \tag{9}$$

This falls significantly below the Squire value of $C_R = \pi\tau$ above trim angles of 4-5 deg, as shown in Fig. 2. If Eq. (9) is substituted into Eq. (4), Pierson and Leshnover's result is reproduced, however.

Sedov[7] obtained the same result as Eq. (9), expressed as

$$C_R = \frac{2\pi}{\cos\frac{\tau}{2}+\pi+\tan\frac{\tau}{2}\log\left(\cot^2\frac{\tau}{2}-1\right)} \tag{10}$$

We conclude that Squire's analysis applies only to small trim angles—as he states in his assumptions—and that a better approximation might be Eq. (9) or (10) divided by Squire's factor $[1+(5/8)(\pi/F_l^2)]$ as an empirical correction for finite fluid weight.

When the plate is not flat the trim angle τ can be measured from zero lift trim, but the total resistance is now $R\beta$ rather than $R\tau$, β being the induced drag angle. Thus Eq. (3) becomes

$$D_w + D_j = R\sin\beta \tag{11}$$

leading to

$$\frac{\delta}{l} = \frac{C_R}{2(1+\cos\theta)}\left[\sin\beta - \frac{C_R\cos^2\beta}{2F_l^2}\right] \tag{12}$$

where θ is the jet angle above the horizontal.

Since methods exist for calculating the two-dimensional pressure distribution on a cambered plate[6] (Squire's[1] methodology is readily adapted, and Sedov[7] gives results by M.I. Gurevich for a circular arc profile, using Sedov's methodology), it is possible to find the induced drag angle β and hence deduce δ/l from Eq. (11).

Any pressure field (such as that generated in the bubble of a surface effect ship) which does not have local regions of pressure equal to $\frac{1}{2}\rho u_0^2$, will not develop a jet. The same would be true (under the same conditions) of a cambered planing plate which had the same shape as the water surface in a pressure field.

References

[1] Squire, H.B., "The Motion of a Simple Wedge Along the Water Surface," *Proceedings of the Royal Society,* Series A, Vol. 243, 1957.

[2] Pierson, J. D. and Leshnover, S., "An Analysis of the Fluid Flow in the Spray Root and Wake Regions of Flat Planing Surfaces," Institute of Aeronautical Sciences, Sherman M. Fairchild Publication Fund Paper 335, Oct. 1948.

[3] Payne, P. R., "A Simplified Approach to Free-Surface Wave Drag Theory," *Journal of the Royal Aeronautical Society,* Vol. 72, March 1968.

[4] Payne, P. R., "On High Efficiency Planing Surfaces and Their Spray Sheets," *Journal of Hydronautics,* Vol.

[5] Payne, P. R., "Notes on a Flat-Bottomed Planing Surface," Payne, Inc. Working Paper 268-11, Jan. 1980.

[6] Llewelyn, R.P., "The Velocity Distribution on a Polynominal Mean Line and the Converse," *Journal of the Royal Aeronautical Society,* Vol. 68, Jan. 1964.

[7] Sedov. L.I., *Two-Dimensional Problems in Hydrodynamics and Aerodynamics,* Interscience Publishers, New York, 1965.

Wave Focusing and Hydraulic Jump Formation

Wilson C. Chin*
Massachusetts Institute of Technology
Cambridge, Mass.

Introduction

THE analysis of hydraulic jumps in shallow water usually centers on jump conditions obtained by relating the upstream and downstream solutions for mean height and speed through mass and momentum conservation. In this classic approach, a resultant decrease of energy across the discontinuity is assumed to occur through turbulent dissipation (e.g., see Lamb[1]). Recently, this author,[2] expanding on some work of Benjamin and Lighthill,[3] generalized the foregoing treatment to allow for the addition of finite amplitude waves. Four quantities, namely, the mean height h, the mean speed U, the wave energy density E, and the wave number k now appear as unknowns; upstream and downstream conjugate solutions were obtained by conserving total mass, total momentum, total energy, and wave crest number. The results of the author's study showed how, in a nondissipative system, the usual loss of mean flow energy could be attributed to the appearance of enhanced downstream radiating waves characterized by significantly increased energy density and wave number.

The exact role played by these waves in setting up the discontinuous flow, however, is unclear; certainly, these "unstable" waves, which experience phenomenal growth through the shock layer, must play some dominant part in a very interesting dynamical process which unfortunately cannot be uncovered by studying jump conditions alone. This Note provides an exploratory study on hydraulic jump for-

Received Jan. 7, 1980.

Index category: Hydrodynamics.

*Research Staff, Dept. of Aeronautics and Astronautics. (Currently Engineering Manager, Pratt and Whitney Aircraft Group, East Hartford, Conn.) Member AIAA.